I J R B A T, Issue (XI) Vol (II) May 2023 : 41-54
A Double-Blind Peer Reviewed & Refereed Journal

e-ISSN 2347 – 517X
Original Article

INTERNATIONAL JOURNAL OF RESEARCHES IN BIOSCIENCES, AGRICULTURE AND TECHNOLOGY

TRANSIENT RESPONSE OF THERMOELASTIC FIELD IN AN ELASTIC PLATE WITH AN EDGE CRACK WITHIN THE FRACTIONAL-ORDER THEORY

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Communicated :03.09.2022 | Revision: 16.09.2022 & 22.09.2022 Accepted: 08.10.2022 | Published: 30.01.2023

ABSTRACT:
This paper considers a transient thermoelastic problem in an isotropic homogeneous elastic plate subjected to thermal load within the fractional-order theory framework during analysis. The uniform-type surface temperature is on the plate's top face, while the bottom face is kept at zero. In order to solve the basic governing equations, an integral transformation was taken into consideration. The thermoelastic behaviours in a plate with an edge crack are investigated. With this solution, the stress intensity factors at the crack tip are numerically calculated through the weight function method. The results are illustrated by numerical calculations considering the material to be an aluminium-like medium and presented graphically.

Keywords :- fractional calculus, non-Fourier heat conduction, thermal stress, integral transform approach, fractional calculus, integral transform.

INTRODUCTION :

In many engineering fields, Solid objects are widely used as structural components. Seismic, mechanical, hydrodynamic, blast, aerodynamic, and thermal loads may be applied to such structural elements. Engineers and scientists worldwide are striving to build cost-effective, reliable structures. Many academics have studied a solid object with varying boundary and loading circumstances using linear theory. Stress analysis also affects systems with mechanical and thermal loads.

Science and engineering are increasingly using fractional-order equations to model dynamical systems that describe various substances' memory and heredity qualities. Fractional calculus will impact classical analysis, linear and nonlinear functional analysis, ordinary and partial differential equations, optimization theory, control theory, and others; see Hilfer [1], Sherief et al. [2], and Tenreiro [3]. The various fractional derivatives and integrals theory emerged in the second half of the 19th century. An excellent description is given by Podlubny [4], Kaczorek [5–6], Sherief and El-Latief [7–9], Siedlecka and Kukla [8–9], Abbas [9–10], Xiong and Niu [10–10], Mahakalkar et al. [11–12], Mittal and Kulkarni [13–13], Hussein [13], and others.

Today, fractional differential equations are used in physics, chemistry, biology, engineering, finance, and other fields. Some of the application of fractional order derivatives includes Diffusion processes [14,15], mechanics of materials [16,17], combinatorics [18,19], inequalities [20], analysis [21], calculus of variations [22–27], signal processing [28], image processing [29], advection and dispersion of solutes in porous or fractured media [30], modelling of viscoelastic materials under external forces [31], bioengineering [32], relaxation and reaction kinetics of polymers [33], random walks [34], and so on. Thus, mathematical modelling of real-life problems generally yields fractional

http://doi.org/10.29369/ijrbat.2023.02.1.0008

differential equations and other difficulties involving special functions of mathematical physics and their extensions and generalization in one or more variables. Bagley and Torvik [42] employed fractional-order derivatives to represent damping forces with memory effect. Wang and Hu [43] observed that the fractional-order derivative term between 0 and 2 always serves as a damping force in single-degree-of-freedom fractional-order vibration systems. Fractional calculus has modified several physical process models. Fractional derivatives were first used by Abel [44] to solve an integral equation in the tautochrone problem formulation. Caputo [45, 46] and Caputo and Mainardi [47, 48] used fractional derivatives and found good agreement with empirical viscoelastic material descriptions.

Of most recent literature, some authors have undertaken work on fractional derivatives in thermoelastic analysis, which can be summarised as follows: Povstenko [49] proposed a quasi-static uncoupled thermoelasticity theory based on the heat conduction equation with a time-fractional derivative of order *a*. Povstenko [50] developed diffusive stress theory using the time-fractional diffusion equation. Cauchy and source problems were discussed. Povstenko [51] examined the temperature distribution and thermal strains in an infinite medium with a spherical cavity using a quasi-static uncoupled theory of thermoelasticity based on the heat conduction equation with a time-fractional derivative. Youssef and Al-Lehaibi [52] used heat conduction in deformable bodies and the Riemann–Liouville fractional integral operator to create a new theory of fractional order generalized thermoelasticity. Youssef and Al-Lehaibi [53] created a half-space-filling elastic material model with constant parameters. Fractional order generalized thermoelasticity theory governed the equations. Laplace transforms, and state space methods will be utilized to solve any boundary condition for the quiescent medium. Sherief et al. [54] established a fractional calculus based on coupled and generalized thermoelasticity theory with one relaxation time. Ezzat and El-Karamany [55] tried to apply these results to two-temperature thermoelasticity with a magnetic field. Youssef and Al-Lehaibi [56] created a cylindrical hollow elastic material mathematical model using fractional order generalized thermoelasticity theory. Povstenko [57] researched axisymmetric thermal stresses in a cylinder by utilizing the heat conduction equation in conjunction with the Caputo time-fractional derivative of order 0. Similarly, using a fractional-order standard thermoelasticity model, Youssef [58] created an elastic half-space with constant elastic parameters. A two-temperature generalized thermoelasticity for fractional-order heat conduction was established by Sur and Kanoria [59].

Youssef [60] investigated the two-temperature thermoelasticity theory of fractional order to a half-space filled with an elastic material with constant elastic parameters under a constant-velocity moving heat source. Youssef et al. [61] created a cylindrical nano-beam mathematical model with constant elastic constants and fractional-order heat conduction. Wang et al. [62] investigated thermoelastic phenomena involving thermal inertia during macro- and microscale heat conduction. Bhattacharya and Kanoria [63] found the two-temperature thermal-elastic-diffusion interaction inside a spherical shell in fractional order generalized thermoelasticity using the Integral transform method. Zenkour and Abouelregal [64] obtained the thermoelastic displacement, stress, conductive temperature, and thermodynamic temperature in a spherical cavity infinite isotropic elastic body. Youssef [65] developed a fraction-order thermoelasticity theory that modifies Duhamel-stress-strain Neumann's

http://doi.org/10.29369/ijrbat.2023.02.1.0008

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ratio. Bachher [66] considered a one-dimensional problem for a homogeneous and isotropic thermoelastic infinite porous material with a reference temperature-dependent modulus of elasticity and thermal conductivity subjected to periodically varying heat sources in the context of fractional order generalized thermoelasticity with one relaxation time parameter.

Santra et al. [67] introduced the three-dimensional generalized thermoelastic coupled issue for a homogeneous isotropic and thermally conducting medium under rotation in fractional order generalized thermoelasticity. Yadav et al. [68] used generalized thermoelasticity with fractional order strain to examine one-dimensional disturbances in a viscoelastic solid with a moving internal heat source and mechanical stress. Green-Naghdi thermoelasticity with energy dissipation is the issue. Bassiouny and Abouelnaga [69] studied the thermoelastic characteristics of a sandwich structure with three piezoelectric layers using fractional order two-temperature generalized thermopiezoelasticity. Gupta and Das [70] used Laplace transform and the eigenvalue approach to solve the deformation of an unbounded transversely isotropic material on fractional order generalized thermoelasticity. Sheoran and Kundu [71] reviewed relevant material to demonstrate fractional calculus's function in thermoelasticity. This review covers traditional heat conduction equation generalizations and fractional thermoelasticity ideas.

Abbas [72] examined the temperature, displacement, and stresses caused by thermal shock loading on the inner surface cavity in an infinite medium with a cylindrical hollow within fractional order generalized thermoelasticity theory. Bachher and Sarkar [73] explored the magneto-thermoelastic response of a homogeneous isotropic two-dimensional rotating elastic half-space solid using generalized thermoelasticity based on the Caputo time-fractional derivative. Povstenko et al. [74] investigated thermal stress regulation in an infinite cylindrical body using the time-fractional heat conduction equation with the Caputo derivative $0 < \alpha - 2$ specified the temperature distribution. Xiong and Niu [76] devised fractional-order thermoelastic diffusion for anisotropic and linear diffusive media [75]. The dynamic behaviour of a semi-infinite medium with one end exposed to thermal and chemical potential shocks was analyzed using the Laplace transform. Chirilă and Marin [76] worked on dipolar thermoelastic materials, a specific instance of multipolar continuum mechanics. Abbas [77] investigated the fractional order derivative affected a two-dimensional thermal shock problem with weak, normal, and strong conductivity under fractional order derivative using Laplace and exponential Fourier transforms with eigenvalues. Lata [78] explored the thermal response for a homogeneous isotropic thick circular plate in the framework of the two-temperature thermoelasticity theory. Mondal et al. [79] studied transient phenomena for a fibre-reinforced media with a cylindrical cavity under an induced magnetic field in the three-phase-lag model of generalized thermoelasticity using a new derivative of the Caputo–Fabrizio type in the heat transport equation. Mittal and Kulkarni [80] used fractional thermoelasticity in two-temperature theory and investigated the thermal fluctuations in the limited spherical region. Mondal [81] introduced a novel mathematical model to examine transient phenomena in a rod in the Lord–Shulman thermoelastic framework based on Eringen's nonlocal elasticity. Heat-insulated rod ends are fastened using Laplace transform.

In this paper, based on time-fractional equations, a transient heat conduction model is developed to study the thermoelastic response in a cracked plate. The Laplace transform and the

http://doi.org/10.29369/ijrbat.2023.02.1.0008

e-ISSN 2347 – 517X
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finite Fourier sine transform are applied to solve the fractional equations. With the aid of the Mittag-Leffler function, the analytical solution is obtained. The weight function method conveys the stress intensity factor at the edge crack's tip. The effects of the temperature field, moisture field, stress response, and stress intensity factors are discussed.

Basic assumptions and governing equations :

For our investigation, a time-fractional thermoelasticity hypothesis was used. We take into account the thermal influence on elastic stresses and deformation; on the other hand, elastic deformation does not affect temperature. The mathematical formulation for the time-fractional heat conduction equation is expressed:

i. The classical Fourier's law of heat conduction [82]

$$q(x,t) = -k\nabla T(x,t) \tag{1}$$

in which $q(x,t)$ is the heat flux vector represents heat flow per unit time per unit area of the isothermal surface, t is the time, x is the position of any point on solid, k is the thermal conductivity, ∇ is the gradient operator, and T is the temperature gradient. It is a vector normal to the surface, respectively. Since the heat flux points to decreasing temperature, the minus sign makes the heat flow a positive quantity. When the heat flux is in W/m^3, and the temperature gradient is in ^{o}C/m, the thermal conductivity has W/(m^{o}C). The main drawback of the traditional Fourier's law is that it results in a parabolic equation for the temperature, which causes thermal waves to propagate at an infinite rate and is, therefore, unsuitable in its current form.

ii. Maxwell-Cattaneo introduced single-phase-lag to evade the discrepancy between the mathematical model [83,84] and the experimental observations [85], and this extension turns the parabolic into a hyperbolic equation as

$$q(x,t) + \tau_0 \frac{\partial q(x,t)}{\partial t} = -k\nabla T(x,t) \tag{2}$$

As a limiting case $\tau_0 \to 0$, one recovers the classical Fourier's law with an infinitely fast propagation. Here the flux relaxes with some given characteristic time constant τ_0, the heat flux's phase lag or so-called relaxation time. Consequently, the propagation velocity is finite.

iii. Recently, a kind of generalization of Eq. (2) consisting of replacing the classical integer-order derivative with fractional order can be referred to in literature [86] and the reference therein.

$$q(x,t) + \tau_0 \frac{\partial^\alpha q(x,t)}{\partial t^\alpha} = -k\nabla T(x,t) \tag{3}$$

with the solution [87] as

$$q(x,t) = -\frac{k}{\tau_0}\int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left[-\frac{(t-\tau)^\alpha}{\tau_0}\right]\nabla T(x,\tau)\,d\tau \tag{4}$$

in which the fractional Caputo derivative of order α with a lower limit zero

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = {}_0^C D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\gamma)^{m-\alpha-1}\frac{\partial^m f(\gamma)}{\partial \gamma^m}d\gamma, m-1<\alpha<m \\ \frac{\partial^m f(t)}{\partial t^m}, \ \alpha = m,\ m\in N \end{cases}$$

(5)

whereas $f(t)$ is a Lebesgue integrable function and the Riemann-Liouville fractional derivative is taken as

$$D_{RL}^\alpha f(t) = \frac{\partial^m}{\partial t^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\gamma)^{m-\alpha-1} f(\gamma)\,d\gamma\right],\ m-1<\alpha<m$$

(6)

wherein Eq. (3), without losing the generality $\Gamma(1+\alpha)$ appearing in the Taylor series is merged in τ_0 terms, Γ is the gamma function, α is introduced to keep the dimension in order and $\partial^\alpha / \partial t^\alpha$ is the fractional time derivative based on Caputo fractional definition [4].

By combining Eq. (3) with the continuity equation, which is given as

$$-\rho C_v \frac{\partial T(x,t)}{\partial t} = \nabla \cdot q(x,t) \tag{7}$$

leads to the hyperbolic heat conduction equation

$$\frac{\partial}{\partial t}\left(1+\tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)T(x,t) = \kappa\Delta T(x,t) \tag{8}$$

http://doi.org/10.29369/ijrbat.2023.02.1.0008

in which $\kappa = k / \rho C_v$ is the thermal diffusivity coefficient, ρ is the density, C_v is the calorific value and $\Delta = \nabla^2 = \nabla \cdot \nabla$ is the gradient operator, respectively.

For the limiting case:

(i) Taking $\tau_0 = 0, \alpha = 0$, Eq. (8) reduces to classical Fourier heat conduction,

(ii) Taking $\alpha \in (0,1)$, Eq. (8) is identified as a fractional generalization of the Cattaneo approach,

Formulation of the Problem :

Time fractional heat conduction equation in the single-phase-lag model :

For our investigation, we consider the transient response of the fractional heat conduction in a plate of thickness h that has a crack along one of its edges. It is decided to use the Cartesian coordinate system $O - xyz$, with the plate having an infinite extent in the y and z directions but having a finite extent in the x direction (i.e., $0 < x \le h$).

As seen in Figure 1, the edge fracture situated in the plane $y = 0$ may be found at the coordinates $0 < x < c, -\infty < y < +\infty$ and is perpendicular to the plate's free surface. T_0 denotes the temperature reference at the beginning of the process. Equation (8) can be rewritten in the dimensionless form as a time-fractional heat conduction equation in the single-phase-lag model with its corresponding boundary conditions after dropping primes for convenience.

$$\frac{\partial}{\partial t}\left(1 + \tau_0 \frac{\partial^\alpha}{\partial t^\alpha}\right)(\Theta + \varepsilon e) = \frac{\partial^2 \Theta}{\partial x^2} \tag{9}$$

subjected to conditions

$$\Theta(x,0) = 0, \frac{\partial \Theta(x,0)}{\partial t} = 0 \tag{10}$$

$$\Theta(0,t) = \Theta_1 H(t), \ \Theta(h,t) = \Theta_2 H(t), \tag{11}$$

where Θ_1 and Θ_2 are prescribed heat constants, e is the strain dilatation along the x direction, $H(t)$ is the Heaviside unit step function and the following non-dimensional variables are used

$$x' = c_1 \eta x, \quad y' = c_1 \eta y, \quad u' = c_1 \eta u, \quad t' = c_1^2 \eta t,$$
$$\tau_0' = c_1^2 \eta \tau_0, \ \eta = 1/\kappa = \rho C_v / k,$$
$$\sigma_{ij}' = \sigma_{ij} / \rho c_1^2, \qquad c_1^2 = (\lambda + 2\mu)/\rho,$$
$$\varepsilon = T_0 \gamma^2 / (\lambda + 2\mu) k \eta,$$
$$\Theta = \gamma (T - T_0) / \rho c_1^2, \ \gamma = \alpha_t (2\lambda + 3\mu)$$

with α_t is the coefficients of linear thermal expansion of the material, λ and μ are the Lame constants, T_0 is the reference temperature, respectively.

The thermal stress function :

To calculate the thermoelastic response of the plate with an edge fracture, we will assume that both of the plate's surfaces $x = 0$ and $x = h$, do not experience any traction as

$$\begin{aligned} &\sigma_{xx}(0,t) = 0, \sigma_{xx}(h,t) = 0, \sigma_{xy}(0,t) = 0, \\ &\sigma_{xy}(h,t) = 0, \sigma_{xz}(0,t) = 0, \sigma_{xz}(h,t) = 0. \end{aligned} \tag{12}$$

Now, suppose the temperature $\Theta = \Theta(x,t)$ is the excess of temperature over Θ_0, the absolute temperature of the plate in a state of zero stress and strain; then, the thermal stress $\sigma = \sigma_{yy}(x,t)$ is connected with u and Θ by the relation

$$\sigma = e - \omega \Theta \tag{13}$$

where quantity $\omega = T_0 \gamma / (\lambda + 2\mu)$, E denotes Young's modulus, α_t the linear expansion coefficient, and ε_{yy} is the strain component which can be obtained using compatibility condition $\partial^2 \varepsilon_{yy} / \partial x^2 = 0$ that gives

$$e = \varepsilon_{yy}(x) = C_1 x + C_2 \tag{14}$$

http://doi.org/10.29369/ijrbat.2023.02.1.0008

where C_1 and C_2 are coefficients to be determined from the boundary conditions of the plate structure. Thus, for the thermoelastic medium in plane strain, using generalized Hooke's law, the thermal stress in the absence of crack, as shown in equation (13), can be rewritten as

$$\sigma = C_1 x + C_2 - \omega\Theta \tag{15}$$

The Solution to the Problem

Following Liang et al. [88], if $\alpha > 0$, $n = [\alpha]+1$, and functions $f(t)$, $f'(t)$, $f''(t)$, ... $f^{(n-1)}(t)$ are continuous in $[0,\infty)$ and of exponential order, while ${}^{C}D_0^{\alpha} f(t)$ with order α is piecewise continuous on $[0,\infty)$, then Laplace transform of Caputo fractional derivative of $f(t)$, is defined as follows

$$L[{}^{C}D_0^{\alpha} f(t)] = s^{\alpha} L[f(t)] - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \tag{16}$$

In view of the above theorem, assuming

$$f(x,0) = \frac{\partial}{\partial t} f(x,0) = \frac{\partial^2}{\partial t^2} f(x,0) = ... = 0. \tag{17}$$

Using Eqs. (13) and (14), applying the Laplace transforms to the Eqs. (9) and (11), bearing Eq. (15) in mind, one obtains

$$(s + \tau_0 s^{\alpha+1})[\bar{\Theta} + \varepsilon(C_1 x + C_2)] = \frac{\partial^2 \bar{\Theta}}{\partial x^2} \tag{18}$$

subjected to conditions

$$\Theta(0,s) = \Theta_1 / s, \ \Theta(h,s) = \Theta_2 / s, \tag{19}$$

where s is the parameter and $\bar{f}$ stands for the Laplace transform of *f*, respectively.

To obtain the solution to Eq. (18), we recall the following property of the finite Fourier sine transform in the domain $0 \le x \le h$

$$F\left\{\frac{\partial^2 \bar{f}(x,s)}{\partial x^2}\right\} = -\xi_n^2 \bar{\bar{f}}(\xi_n, s) + \xi_n [\bar{f}(0,s) - (-1)^n \bar{f}(h,s)] \tag{20}$$

where $\bar{\bar{f}}$ stand for the finite Fourier transform of $\bar{f}$, and $\xi_n = n\pi / h$, $k = 1,2,...$ respectively.

Performing the finite Fourier sine transform of both sides of Eq. (18) subject to conditions (19), one obtains

$$\bar{\bar{\Theta}}(\xi_n, s) = \frac{\Theta_{12}\xi_n^2 - \Psi\varepsilon\Omega}{s\xi_n(\Psi + \xi_n^2)} \tag{21}$$

where $\Theta_{12} = \Theta_1 - (-1)^n \Theta_2$, $\Psi = s + \tau_0 s^{\alpha+1}$ and $\Omega = C_1(-1)^{n+1} h + C_2[1-(-1)^n]$.

Then we perform the inverse finite Fourier sine transform to both sides of Eq. (21) as

$$\bar{\Theta}(x,s) = 2\sum_{n=1}^{\infty}\left[\frac{\Theta_{12}\xi_n^2 - \Psi\varepsilon\Omega}{s\xi_n(\Psi + \xi_n^2)}\right]\sin(\xi_n x) \tag{22}$$

Using Eqs. (16) and (17), applying the Laplace transforms to the dimensionless governing Eq. (15), the transformed equations are given as

$$\bar{\sigma} = C_1 x + C_2 - \omega\bar{\Theta}(x,s) \tag{23}$$

If the plate is only subjected to thermal shock without constraint along its boundaries, then the unknown constants C_1 and C_2 can be solved from the following conditions

$$\int_0^h \bar{\sigma}_{yy}(x,s)dx = 0, \ \int_0^h \bar{\sigma}_{yy}(x,s)x dx = 0, \tag{24}$$

which may be used to determine two constants C_1 and C_2. Thus, Eqs. (22)–(24) describe the analytical solutions of thermal parameters $\bar{\Theta}$ and $\bar{\sigma}$, respectively, in the Laplace domain.

The thermal stress intensity factor :

Following [89,90], the crack problem considered, to ensure crack faces to be free we require that an equal and opposite axial stress will be superposed. Using the weight function method, the stress intensity factor (SIF) K_I near the edge crack tip can be calculated by the following integral [90]:

$$K_I = \frac{2\sqrt{h}}{\sqrt{\pi c}(1-c)^{3/2}} \int_0^c \frac{\bar{\sigma}_{yy}(x,s) F_1(x,c)}{\sqrt{1-(x/c)^2}} dx \tag{25}$$

Here, $c' = c_1 \eta c$, $F_1(x,c)$ is a non-dimensional weight function [89] is given as

$$F_1(x,c) = f_1(c) + f_2(c)\left(\frac{x}{c}\right) + f_3(c)\left(\frac{x}{c}\right)^2 + f_4(c)\left(\frac{x}{c}\right)^3,$$

http://doi.org/10.29369/ijrbat.2023.02.1.0008

where

$f_1(c) = 0.46 + 3.06c + 0.84(1-c)^5 + 0.66c^2(1-c)^2,\ f_2(c) = -3.52c^2,$
$f_3(c) = 6.17 - 28.22c - 34.54c^2 - 14.39c^3 - (1-c)^{1/2} - 5.88(1-c)^5 - 2.64c^2(1-c)^2,$
$f_4(c) = -6.63 + 25.16c - 31.04c^2 + 14.41c^3 + 2(1-c)^{1/2} + 5.04(1-c)^5 + 1.98c^2(1-c)^2.$

It is noted that the above integral computation is effective for a positive (tensile) stress since a negative (compressive) stress does not give rise to crack opening but closing. Of course, from another point of view, a negative stress intensity factor may be understood as a shield effect to prevent the crack from advancing.

The numerical inversion of the Laplace transforms :

Consider the Gaver-Stehfest algorithm [91-93], which aims to approximate $f(t)$ by a sequence of function, can be given as

$$f(t) \approx f_n(t) = \left[\frac{1}{t}\text{In}(2)\right]\sum_{n=1}^{L} a_n F\left[\frac{n}{t}\text{In}(2)\right], \qquad n \geq 1, \quad t > 0,$$

(26)

where $F[.]$ is the Laplace transform of $f(t)$.

The coefficients a_n depend only on the number of expansion terms n, defined as

$$a_n = (-1)^{n+L/2} \sum_{k=[(n+1)/2]}^{\min(n,L/2)} \frac{k^{L/2}(2k)!}{(L/2-k)!k!(k-1)!(n-k)!(2k-n)!},$$
$$n \geq 1,\ 1 \leq L \leq n \qquad (27)$$

The convergence of Gaver-Stehfest algorithm for numerical inversion of the Laplace transform was developed by Kuznetsov [94]. It is well proved that the approximations $f_n(t)$ converge to $f(t)$, if f is continuous at t and of bounded variation in a neighbourhood of t. It has been mathematically demonstrated that if a significant number of terms from the sequence are considered, the series will converge. Because the thickness of the plate is rather thin, the solution that has been presented here will definitely converge. To summarise, our convergence claim states that if we consider a sufficient number of different terms, the solutions to the series will eventually converge on the precise answer, and the margin of error will tend to be zero across the board. To put it another way, if we make the phase lower and smaller, the convergence rate will be significantly increased. To meet the convergence of the infinite series in the solution and the conditions to be imposed with the functions at an arbitrary point, we must roughly replace $\sum_\infty$ in the temperature and its stresses by $\sum_{20}$.

CONCLUSION :

The proposed closed-form for a transient thermoelastic problem in an isotropic homogeneous elastic plate subjected to thermal load within the fractional-order theory framework is considered during analysis. In order to solve the basic governing equations, an integral transformation technique was taken into consideration. The thermoelastic behaviours in a plate with an edge crack are investigated.

NOMENCLATURE :

λ, μ Lamè constants
t Time
ρ Density
C_v Specific heat at constant strain
T Temperature
T_0 Reference temperature
a_t Coefficient of linear thermal expansion
σ_{ij} Components of stress tensor
u_i Components of displacement vector
e Dilatation
λ Thermal conductivity
κ Diffusivity
τ_0 Relaxation times
ε Thermal coupling parameter

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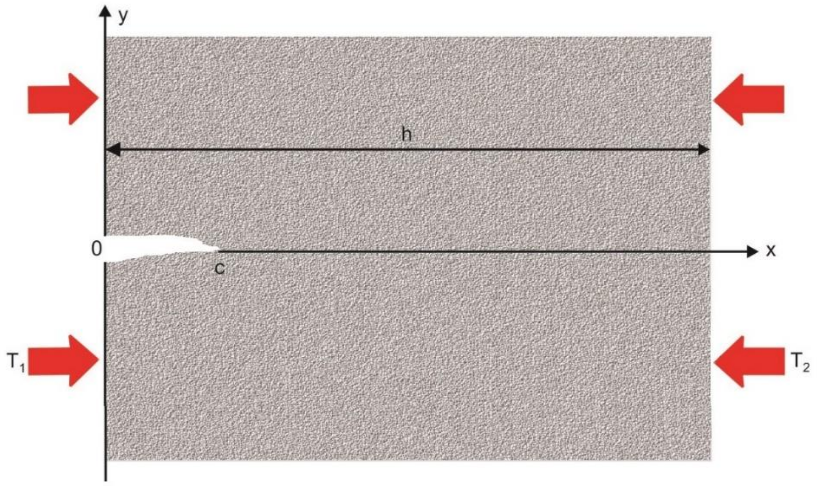
Figure 1: infinite plate with a crack at its edge subject to uniform thermal loading



http://doi.org/10.29369/ijrbat.2023.02.1.0008